# Gabor and Morlet Filter Bank Implementation

July 21, 2011

This is to explain the parameters in our implementation of gabor and morlet filter bank.

## 1 Gabor filters

### 1.1 One dimensional case

The 1d-gabor wavelet is defined spatialy by

$$\psi(x) = \exp\left(-\frac{x^2}{2\sigma^2} + i\xi x\right)$$

Its fourier transform is

$$\hat{\psi}(\omega) = \sigma \exp\left(-\frac{\sigma^2(\omega - \xi)^2}{2}\right)$$

Its value in 0 is $\hat{\psi}(0) = \exp(-\sigma^2\xi^2/2)$ so that we have

$$\xi\sigma = \sqrt{-2\log(\hat{\psi}(0))} \tag{1}$$

We compute the wavelets

$$\psi_j = \frac{1}{a^j\sigma}\psi\left(\frac{x}{a^j}\right)$$

where $a$ is the scale factor.

If we call $\tau$ the value of the two gabor where there plots intersect, we have, as we can see on figure 1:

$$\frac{\xi}{a} + \frac{g^{-1}(\tau)}{a\sigma} = \xi - \frac{g^{-1}(\tau)}{\sigma}$$

where $g(x) = \exp(-x^2/2)$ ie $g^{-1}(\tau) = \sqrt{-2\log(\tau)}$ so we have

$$\xi\sigma = \sqrt{-2\log(\tau)}\frac{a+1}{a-1} \tag{2}$$

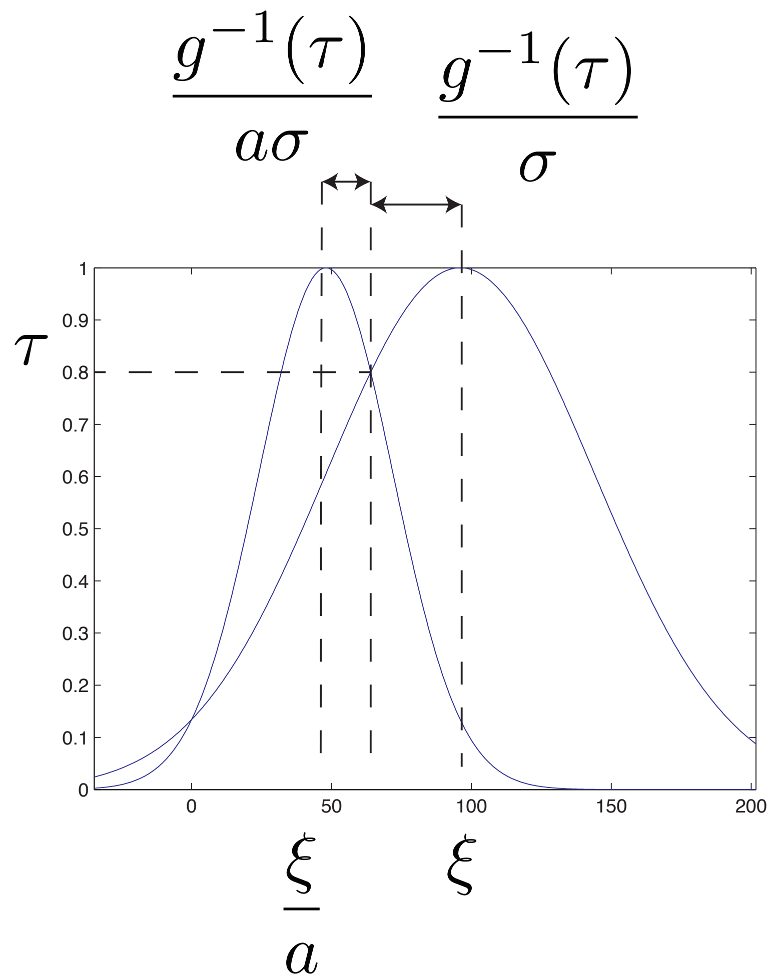




Figure 1: fourier transform of adjacent scale gabor wavelet. $\tau$ has been set to 0.8 here

The value of $\xi$ is fixed by the fact that we need the hig frequency information so we set

$$\xi = 3\pi/4$$

Equation 1 and 2 show that the choice of $\sigma$ is a trade off between two antagonist requirement on the wavelet :

1. a zero - mean : $\sigma$ should be large.
2. a good littlewood paley sum for the filter bank and for that $\tau$ should be large so $\sigma$ shoud be small

If we use 1 and 2 we can see that

$$\hat{\psi}(0) = \tau^{\frac{(a+1)^2}{(a-1)^2}}$$

so we can see that these tow requirement are more and more compatible as we take more and more band per octave.

In the implementation, the only parameter about the wavelet that the user can set is $\boxed{\tau_{as} = \tau}$. 'as' stands for adjacent scales. The implementation of parameters is the following:

1. The value of $\xi$ is set to $3\pi/4$.
2. The user choses values for $\tau$, $a$ and $J$
3. The value of $\sigma$ is computed with : $\sigma = \frac{\sqrt{-2\log(\tau)}}{\xi} \frac{a+1}{a-1}$

There is another parameter called $\boxed{\tau_{lc}}$. 'lc' stands for low-coarse. It controls the value at crossing between the low pass filters $\phi_J$(a gaussian) and the coarsest scale wavelet $\psi_{J-1}$ and this parameters determines the value of the bandwith of the low pass filter in the same spirit.

### 1.2 Two dimensional case

In the 2d case we simply take a rotated-slanted 2-d gabor filters with parameters $\sigma$ and $\xi$ determined as in the one dimensional case :

$$\psi_\theta(\mathbf{x}) = \exp\left( - \frac{\mathbf{x}^T R_{-\theta} \begin{pmatrix} 1 & 0 \\ 0 & slant^2 \end{pmatrix} R_\theta \ \mathbf{x}}{2\sigma^2} + i\xi(1,0).(R_\theta \mathbf{x}) \right)$$

The parameter *slant* is adapted to the number of orientation $L$ :

$$slant = \frac{4}{L}$$

## 2 Morlet filters

Morlet filters are modified gabor filters that have zero-mean : The idea is to substract to a gabor its envelop times a constant so that the results has zero mean :

$$\psi(x) = \exp\left(-\frac{x^2}{2\sigma^2}\right)\left(\exp(i\xi x) - \exp(-\sigma^2\xi^2/2)\right)$$

Its fourier transform is

$$\hat{\psi}(\omega) = \sigma\left(\exp\left(-\frac{\sigma^2(\omega-\xi)^2}{2}\right) - \exp\left(-\frac{\sigma^2(\omega^2+\xi^2)}{2}\right)\right)$$

and we can see it has zero mean $\hat{\psi}(0) = 0$

Two dimensional morlet filters are implemented with

$$\psi_\theta(\mathbf{x}) = \exp\left(-\frac{\mathbf{x}^T R_{-\theta}\begin{pmatrix}1 & 0\\ 0 & slant^2\end{pmatrix}R_\theta\,\mathbf{x}}{2\sigma^2}\right)\left(\exp(i\xi(1,0).(R_\theta\mathbf{x}))) - \exp(-\sigma^2\xi^2/2)\right)$$

As we can see on figure 2 there behavior is approximately the same in the domain of large response, we use the same normalization and parameter choice than with gabor wavelet.

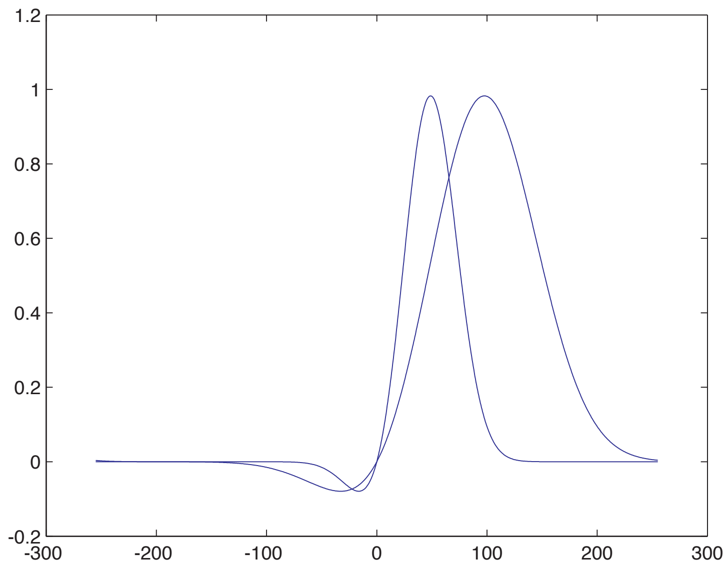




Figure 2: fourier transform of adjacent scale morlet wavelet with the same parameters as in 1